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On the Proper Motion of Sirius in Declination.

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1. In the Astronomische Nachrichten Nos. 514, 515, 516, Bessel has shown that the proper motion of Sirius in right-ascension is not uniform; and in the same Journal Nos. 745, 746, 747, Dr. C. A. F. Peters has represented these anomalies by the hypothesis that Sirius revolves around a centre at some distance from itself with the following elements;

Passage of the Lower Apside	1791.431
Mean Yearly Motion	$7^\circ.1865$
Eccentricity	0.7994

and the difference (q) between the deviation of Sirius relatively to α and β Orionis from the Tabulae Regionontanae is expressed by the formula

$$q = 0^s\,127 + 0^s.00050\,(t - 1800) + 0^s.171\,\sin\,(u + 77^\circ\,44').$$

Tables of q have been given by Wolfers in the Tabulae Reductionum.

2. The hypothesis mentioned above, rests with regard to evidence, solely upon variations in right-ascension. So far as I know, no attempt has yet been made to apply it to the declinations of Sirius. It is, however, manifest that if the elements cited shall be found to represent actually existing deviations in declination, the hypothesis will receive a strong confirmation.

3. That such deviations in declination do really exist in the case of Sirius would be now generally admitted. The first person who seems to have noticed them is my friend Mr. E. Schubert, in the Astronomical

Journal vol. I. p. 154. Since that time the same thing has been mentioned by Dr. C. A. F. PETERS, Astr. Nachr. XXXII, 51 ; by the Rev. R. MAIN (Monthly Notices of the Royal Astronomical Society for March 1860,) and by others. The discrepancies are sufficiently remarkable at first sight.

4. The observations which I shall use are those of BRADLEY [Fundamenta Astronomiæ auctore Bessel], BESSEL 1820, POND 1822, STRUVE 1824, ARGELANDER 1830, TAYLOR [Madras General Catalogue], HENDERSON [at the Cape, 1833], MACLEAR [at the Cape, 1834], AIRY [Cambridge], HENDERSON [Edinburg], AIRY [Greenwich Twelve-Year Catalogue, Six-Year Catalogue, Results since 1854], BUSCH [Astr. Nachr. No. 422], BESSEL 1843 [with the Repsold meridian-circle at Königsberg], and finally MOESTA. I have been reluctantly compelled to omit the consideration of Maskelyne's, and Pond's observations [except those of 1822, reduced by Olufsen], because they were both in the habit in their own reductions of using Bradley's thermometer coefficient for refraction; which is known to be quite erroneous; so that without a new reduction it would be very difficult to derive accurate results in case of a star culminating so low at Greenwich as Sirius.

Piazzi's catalogue too needs a new and more careful reduction, before any results worthy of confidence in so delicate a matter could be drawn from them.

5. It seems preferable to employ relative declinations in this investigation. As however the fundamental stars south of the equator are few and scattered, it will be necessary to scrutinize the reductions quite carefully.

The observations which I have mentioned were partly made in the southern hemisphere and at Madras, where Sirius culminates nearer the zenith, than in Europe. Provided the temperature correction for refraction is rightly assumed, observations made north and south of the equator will not differ by a constant amount from this cause.

6. As stars of comparison I have employed the following:

α Virginis,

2 α Librae [or α^2 Librae],

α Scorpii,

2 α Capricorni [or α^2 Capricorni].

The differences between observed declinations of the five stars, and those computed in the Tabulae Regiomontanae, in the sense O—C, are contained in the following table; where q' denotes this quantity for Sirius less the mean of those for the four stars.

	Pond 1822 I.	Pond 1822 II.	Struve 1824	Argelander 1830	Taylor 1835	Henderson 1833	Airy 1834	Maclear 1834	Henderson 1837	Airy 1840
α Canis majoris . .	+1″.68	+0″.33	+0″.75	+0″.97	+0″.48	—0″.40	+0″.82	—0″.11	+0″.82	+0″.59
α Virginis	3.95	1.29	2.09	1.88	+2.19	+1.19	2.95	+0.40	1.65	+0.97
2 α Librae	4.82	0.45	1.53	2.04	—0.04	0.34	2.09	—0.32	0.70	—0.23
α Scorpii	4.88	1.12	3.14	2.75	+1.93	0.69	3.42	+0.87	1.53	+1.28
2 α Capricorni . .	——	+2.90	2.78	+2.69	+4.57	+2.08	+4.35	+2.25	+2.45	+1.90
Mean for 4 Stars.	——	+1.44	+2.38	+2.34	+2.16	+1.08	+3.20	+0.80	+1.58	+0.98
q'		—1″.11	—1″.63	—1″.37	—1″.68	—1″.48	—2″.38	—1″.71	—0″.76	—0.39
Mean Epoch of obs.					1832.5					1838.5

	Busch 1840	Bessel 1843	Airy 1845	Airy 1850	Moesta 1855	Airy 1856
α Canis majoris . .	—0″.07	+2″.80	+3″.39	+3″.80	+1″.98	+3″.15
α Virginis	+0.83	1.99	1.55	1.56	+1.29	+1.95
2 α Librae	—0.73	0.99	0.72	0.72	—0.95	+0.08
α Scorpii	—1.11	2.23	1.34	1.94	+1.90	+2.01
2 α Capricorni . .	+0.80	1.55	2.50	2.73	+2.40	+2.43
Mean for 4 Stars.	—0.05	+1.69	+1.53	+1.74	+1.16	+1.62
q'	—0.02	+1.11	+1.86	+2.06	+0.82	+1.53
Mean Epoch	1838.5		1844.5	1850.8		1856.5

7. I have cited the numbers in the columns 1, 2, 4, 6, 11 from

Bessel's paper in the Astronomische Nachrichten Nos. 514, 515, 516. The other authorities are:

Struve's Positiones Mediae, Introduction. I have employed the corrected data in place of slightly different numbers given by Bessel.

Taylor's Madras General Catalogue. I have had no difficulty in referring these numbers to the epoch 1832.5.

Airy's Cambridge Observations. The numbers deducible from the "First Cambridge Catalogue" are slightly different.

Maclear's Cape Observations 1834. I do not know whether other volumes of this series have appeared. Sir Thomas Maclear has observed Sirius since, and doubtless the comparison-stars too.

Henderson's Edinburg Observations. The numbers which I have given are deduced from Wolfers' Tabulae Reductionum, p. XLV, there reduced to 1830.

Airy's Twelve- and Six-Years' Catalogues furnished the numbers for 1840, 1845, 1850.

Bessel 1843 is deduced from the numbers in the Tabulae Reductionum, l.c; Moesta 1855 from his "Observaciones Astronomicas."

Finally, I have compared the "Greenwich Results" for 1854—1858 with Zech's continuation of the Tabulae Regiomontanae. The deviations were found to be

	1854	1855	1856	1857	1858
α Canis majoris	+3″.72	+3″.76	+3″.53	+2″.61	+2″.12
α Virginis	2.15	1.58	+2.52	2.04	+1.47
2 α Librae	0.48	0.44	−0.10	0.09	−0.50
α Scorpii	2.13	2.49	+2.28	1.04	+2.11
2 α Capricorni	+1.77	+2.68	+3.67	+2.49	+1.52

The arithmetical means of these numbers, taken crosswise, are given above as Airy 1856.

8. It must not be forgotten that the elements of reduction employed may have some effect in changing mean places. For instance, the value of the constant of lunar nutation, used to reduce the series, Pond 1822 (in which I have employed Olufsen's reduction), Argelander 1830, Busch 1840, was v. Lindenau's; viz. 8″.977; while 9″.25 was generally used by British astronomers till 1856 inclusive, and Dr. Peters' more correct value $9''.2231 + 0''.0009\left(\frac{t-1800}{100}\right)$ has been employed for the series, Struve 1824, and Bessel 1843. The latter pair of numbers differ much less than the different series do *inter se*, and I shall accordingly neglect the correction arising; but the difference between v. Lindenau's and Dr. Peters' values is too large to be passed over.

9. I have computed the correction arising from this source in the quantity q', and found it equal to

$$0''.41 \sin (\Omega + 276^\circ.1)$$

from which formula the values for 1820, 1822, 1830 and 1838.5 are respectively $-0''.40$, $-0''.37$, $+0''.41$ and $-0'.40$. 1820 is included because that is the epoch of Bessel's observations from which, in connection with Bradley, the Tabulae Regiomontanae were derived.

Bradley's observations reduced by Bessel (using $9''.648$ as the constant of lunar nutation) require a similar correction

$$0''.71 \sin (\Omega + 96^\circ)$$

which for 1755 is equal to $-c''.70$.

The variation of the coefficient of aberration will have much less effect, and one which cannot be calculated without almost a new reduction of the observations in question. Aberration it is well known depends almost entirely upon the apparent time of culmination, which will be much the same on the average for different fundamental stars.

10. We are now enabled to give a series of values of q', which may be nearly relied upon. The following table includes those previously given, with the corrections added, and also the values $-0''.70$ for 1755, and $-0''.40$ for 1820, which derive their origin from the fact that the places of the Tabulae Regiomontanae for 1755 and 1820 were actually observed, but reduced with nutation-constants slightly differing from the definitive results of Dr. Peters.

DATE	q'	DATE	q'
1755	$-0''.70$	1837	$-0''.76$
1820	-0.40	1838.5 Airy	-0.39
1822	-1.48	—— Busch	-0.42
1824	-1.63	1843	$+1.11$
1830	-0.96	1844.5	$+1.86$
1832.5	-1.68	1850.8	$+2.06$
1833	-1.48	1855	$+0.82$
1834 Airy	-2.38	1856.5	$+1.53$
—— Maclear	-1.71		

11. A certain periodicity in these values is sufficiently obvious. In the first place the method of least squares was applied to the whole series upon the supposition of a uniform proper motion. The following correction to the relative declination of Sirius

$$-1''.198 + 2''.512\ (t - 1800)$$

will reduce the sum of the squares of the resulting errors from 32.92 to

25.29. But the periodicity still remains. Bradley's place (Bessel's reduction as modified above) is represented within 1″.63; but it is impossible to conciliate the observations about 1834 with those about 1850; while the resulting probable error of one value of q' is 0″.88, a quantity much larger than would be indicated by the comparative errors of each pair.

12. That Bradley's (or rather Bessel's) place for 1755 is not grossly erroneous, may be inferred from a comparison of Mayer's and Lacaille's catalogues, with the Tabulae Regimontanae. The deviations of these catalogues are

	LACAILLE 1750	MAYER 1756
α Canis majoris	+0″.27	+3′.84
α Virginis	+1.65	+1.62
2 α Librae	+2.87	+6.18
α Scorpii	—0.78	+4.76
2 α Capricorni	—4.44	+5.06
Mean of 4 Stars	—0.17	+4.40
q'	+0.44	—0.56

13. I shall now introduce unknown quantities depending on the hypothesis suggested by Bessel and applied by Dr. Peters to the deviations of Sirius in right-ascension. As my object is not to deduce exact elements of the orbit of Sirius, for which purpose a more extended discussion would be necessary, but to test the applicability of Dr. Peters' elements to represent observations which did not serve to deduce them, I shall use at once these elements themselves so far as applicable, and introduce four new constants relating to declination to replace the four which have regard only to right-ascension. Peters' elements, however, are not the most probable elements deducible from our present material; the resulting errors will then be somewhat larger on the whole than they would be if new elements were to be deduced.

14. I shall therefore attempt to represent the values of q' given above by the formula

$$q' = w + x\left(\frac{t-1800}{100}\right) + y \sin u + z \cos.\, u$$

or

$$o = - q' + w + \text{etc.}$$

where u is derived from the formulae

$$M = 7^\circ.1865\ (t - 1791.431) = u - 0.7994 \sin u.$$

15. The resulting numbers are contained in the following tables,

in forming which I have employed Wolfers' Tabula C [Tabulae Reductionum, p. XXXIII].

	M.	u
1755	98°.25	132°.2
1820	205.31	194.1
1822	219.68	202.3
1824	234.06	210.7
1830	277.18	238.3
1832.5	295.14	251.7
1833	298.73	254.6
1834	305.92	260.8
1837	327.48	282.8
1838.5	338.26	297.7
1843	10.61	40.1
1844.5	21.39	61.7
1850.8	66.66	109.8
1855	96.84	131.7
1856.5	107.62	138.4

Equations of Condition.

$$0 = +0''.70 + w - 0.45x + 0.74y - 0.67z$$
$$0 = +0.40 + w + 0.20x - 0.24y - 0.97z$$
$$0 = +1.48 + w + 0.22x - 0.38y - 0.93z$$
$$0 = +1.63 + w + 0.24x - 0.51y - 0.86z$$
$$0 = +0.96 + w + 0.30x - 0.85y - 0.53z$$
$$0 = +1.68 + w + 0.32x - 0.95y - 0.31z$$
$$0 = +1.48 + w + 0.33x - 0.96y - 0.27z$$
$$0 = +2.38 + w + 0.34x - 0.99y - 0.16z$$
$$0 = +1.71 + w + 0.34x - 0.99y - 0.16z$$
$$0 = +0.76 + w + 0.37x - 0.97y + 0.22z$$
$$0 = +0.39 + w + 0.38x - 0.89y + 0.46z$$
$$0 = +0.42 + w + 0.39x - 0.89y + 0.46z$$
$$0 = -1.11 + w + 0.43x + 0.64y + 0.76z$$
$$0 = -1.86 + w + 0.44x + 0.88y + 0.47z$$
$$0 = -2.06 + w + 0.51x + 0.94y - 0.34z$$
$$0 = -0.82 + w + 0.55x + 0.75y - 0.67z$$
$$0 = -1.53 + w + 0.56x + 0.66y - 0.75z$$

The final equations deduced therefrom by the method of least squares are

$$0 = +6''.61 + 17.00w + 5.47x - 4.01y - 4.25z$$
$$0 = +0.13 + 5.47w + 2.57x - 1.26y - 0.75z$$
$$0 = -16.22 - 4.01w - 1.26x + 11.10y + 0.40z$$
$$0 = -4.50 - 4.25w - 0.75x + 0.40y + 5.90z$$

From these by the usual process of elimination were deduced

$$w = -0''.56;\ x = +2''.02;\ y = +1''.47;\ z = +0''.51$$

that is

$$q^r = 0''.56 + 0''.020\ 2\ (t - 1800) + 1''.47 \sin w + 0''.51 \cos. n.$$

16. The sum of the squares of the final residuals ([*nn*. 4]) is found to be 3.28 that is a little more than *one-eighth* of the least possible amount on the hypothesis of uniform motion.

Substituting these values of the unknown quantities in the equations we obtain as residuals the following:

1755	$-0''.02$	1837	$-0''.37$
1820	-0.60	1838.5	-0.47
1822	$+0.33$	1838.5	-0.44
1824	$+0.36$	1843	$+0.53$
1830	-0.51	1844.5	$+0.01$
1832	$+0.21$	1851	-0.38
1833	$+0.04$	1855	$+0.49$
1834	$+0.97$	1856	-0.37
1834	$+0.30$		

17. We may therefore consider two things as proved; 1st, that the motion of Sirius in declination is not uniform; 2d, that the deviations from uniformity are readily explained upon Bessel's hypothesis, that Sirius is as to its visible position not the centre of gravity of its own system; in other words that there is a large invisible mass present in that system.

18. Dr. Peters has shown conclusively that the irregularities of Sirius in right-ascension are explained upon that hypothesis, and the best independent test applicable is the agreement or non-agreement of the declinations with the same explanation. I have shown that there is an essential agreement, and the conclusion cannot, I think, be avoided, "That Sirius revolves about a centre at a very considerable distance from its optical centre."

Observatory of Harvard College, Sept. 20th, 1861.